

On Kharitonov's Theorem without Invariant Degree Assumption

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Abstract

The original statement of Kharitonov's theorem requires that all polynomials in the family have the same degree. It has been shown recently that such an assumption is unnecessary. Here we show that the validity of the stronger statement follows from a classical proof combined with additional elementary considerations.

1 Introduction

The original statement of Kharitonov's theorem [3] requires that all polynomials in the family have the same degree. It has been shown recently [4, 2] that such an assumption is unnecessary. Here we show that the validity of the stronger statement follows from a classical proof (see, e.g. [1]) combined with additional elementary considerations.

2 Statement and proof

Given $q_i^-, q_i^+ \in \mathbb{R}$ with $q_i^- \leq q_i^+$, $i = 0, 1, \dots, n$, and q_n^+ and q_n^- not both zero, consider the interval polynomial family

$$P := \{p(s) = q_n s^n + \dots + q_0 : q_i \in [q_i^-, q_i^+], \quad i = 0, 1, \dots, n\}$$

and the four associated ‘‘Kharitonov polynomials’’

$$K_1(s) := q_n^- s^n + q_{n-1}^- s^{n-1} + q_{n-2}^+ s^{n-2} + q_{n-3}^+ s^{n-3} + q_{n-4}^- s^{n-4} + \dots$$

$$K_2(s) := q_n^- s^n + q_{n-1}^+ s^{n-1} + q_{n-2}^+ s^{n-2} + q_{n-3}^- s^{n-3} + q_{n-4}^- s^{n-4} + \dots$$

$$K_3(s) := q_n^+ s^n + q_{n-1}^+ s^{n-1} + q_{n-2}^- s^{n-2} + q_{n-3}^- s^{n-3} + q_{n-4}^+ s^{n-4} + \dots$$

$$K_4(s) := q_n^+ s^n + q_{n-1}^- s^{n-1} + q_{n-2}^- s^{n-2} + q_{n-3}^+ s^{n-3} + q_{n-4}^+ s^{n-4} + \dots$$

Theorem 1 *All polynomials in P are Hurwitz stable if and only if K_1 , K_2 , K_3 and K_4 are.*

Proof Necessity is obvious. We proceed to prove sufficiency. Since q_n^+ and q_n^- are not both zero, there is no loss of generality in assuming that $q_n^+ > 0$. Stability of K_4 implies that $q_{n-1}^- > 0$ and that of K_1 then implies that $q_n^- \geq 0$. Since $q_n^+ \geq q_{n-1}^- > 0$, stability of the four K_i 's implies that all other coefficients are positive, i.e., $q_i^+ \geq q_i^- > 0$, $i = 0, 1, \dots, n-2$. We now proceed by contradiction. Thus let $\hat{p} \in P$ be unstable. Denote its coefficients by \hat{q}_i , $i = 0, 1, \dots, n$. Since $\hat{q}_n \geq q_n^- \geq 0$, either $\hat{q}_n > 0$ or $\hat{q}_n = q_n^- = 0$. In the former case, define $\hat{k} := K_3$, in the latter case, $\hat{k} := K_1$. In both cases all polynomials in the line segment

$$E := \{(1-t)\hat{p} + t\hat{k} : t \in [0, 1]\}$$

have the same degree (n or $n-1$) and E contains both a stable (\hat{k}) and an unstable (\hat{p}) polynomials. For $\omega \in \mathbb{R}$ let $E(j\omega)$ and $P(j\omega)$ be the value sets associated with E and P . It follows from the Zero Exclusion Condition (see, e.g., Lemma 5.7.9 in [1]) applied to E that, for some ω^* ,

$$0 \in E(j\omega^*) \subseteq P(j\omega^*).$$

Also, since $p_0^+ \geq p_0^- > 0$, it is clear that $0 \notin P(j0)$. The argument used in the proof given in [1], specifically ‘‘case 2’’ p. 78, now applies.¹ The contradiction revealed in that argument completes the proof.

¹The notation used here is essentially the same as that used in [1], with however our $P(j\omega)$ being $p(j\omega, Q)$ in [1].

3 Conclusion

An elementary proof was provided for Kharitonov's theorem without invariant degree assumption.

References

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