Robust Techniques for Visual Motion Estimation

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Abstract

In a vision guided robotic manipulator it is necessary to estimate the motion parameters of the moving object before some control scheme can be implemented. The classical solution to the general problem of state estimation from a sequence of noisy measurements is the Kalman filter which aims at minimizing the covariance of the estimation error. In this paper certain concepts from robust control such as H_∞-synthesis and μ analysis have been adopted to determine the motion parameters of a moving object from a sequence of noisy measurements. [1]

1 Introduction

The estimation of motion parameters from a sequence of images had been an active area of research for the past few years and a number of papers have been published. There are mainly two approaches to solve this problem – the feature based method and optical flow based method. Aggarwal and Nandakumar [1] gives a review of the various methods that have been developed. Azarbayejani and Pentland[2] and Weng *et al*[3] give some of the latest developments in this field. Broida and Chellappa[5],and Wilson[4] have used the Extended Kalman Filter for estimation of motion from a sequence of images. In this paper the successful robust control techniques such as H_∞ and μ analysis will be applied to this nonlinear estimation problem and a feature based technique will be used.

2 The Visual motion Estimation Problem

The system model is taken as a rectangular block undergoing unknown rotational and translational motion ([8]). The motion of the object is described by a constant velocity model written in terms of the object center (X_c, Y_c) and the rotation β, about a horizontal line as shown in figure 1. Only motion in a two dimensional plane is considered and the distance of the camera from the object plane is kept fixed . Define the state vector as :

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} X_c \\ \dot{X}_c \\ Y_c \\ \dot{Y}_c \\ \beta \\ \dot{\beta} \end{bmatrix} \tag{1}$$

The resulting state equation is

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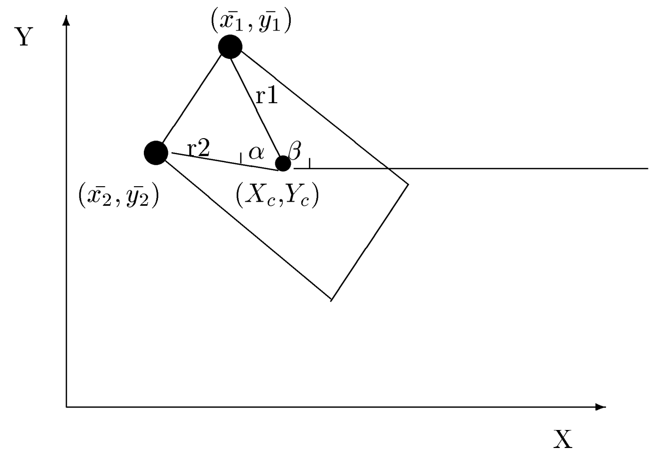

Figure 1: Object Description

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} \tag{2}$$

where w is the disturbance vector. The disturbance vector accounts for any deviation from the constant velocity model assumed such as trajectory perturbations, small random accelerations and small constant accelerations.

The image forming process is described by a perspective transformation as shown in figure 2. It is assumed that the camera coordinate system is aligned with the object coordinate system. Only two feature points are used since the motion of the object is constrained to a two dimensional plane. The feature points used are two corners of the target $(\bar{x_1}, \bar{y_1})$ and $(\bar{x_2}, \bar{y_2})$ as shown in figure 1. It is also assumed that the correspondence problem of extracting the same set of feature points from successive images is solved.

The coordinates of the feature point used on the image plane is expressed in terms of the position and orientation of the object (X_c, Y_c, β) and the known structural parameters (r_1, r_2, α) as

$$\begin{bmatrix} \bar{x}_1 \\ \bar{y}_1 \\ \bar{x}_2 \\ \bar{y}_2 \end{bmatrix} = \lambda/(\lambda - z) \begin{bmatrix} x_1 + r_1 cos(x_5) \\ x_3 + r_1 sin(x_5) \\ x_1 + r_2 cos(x_5 + \alpha) \\ x_3 + r_2 sin(x_5 + \alpha) \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{3}$$

λ is focal length of lens, z is distance of the image plane from the object plane and v is the noise vector representing pixel noise and quantization effects. ;

The object is assumed to have translational velocities in the x and y directions of 2mm/sec and rotational velocity of 10 deg/sec(0.175 rad/sec). The sample period was taken to be 1 sec.

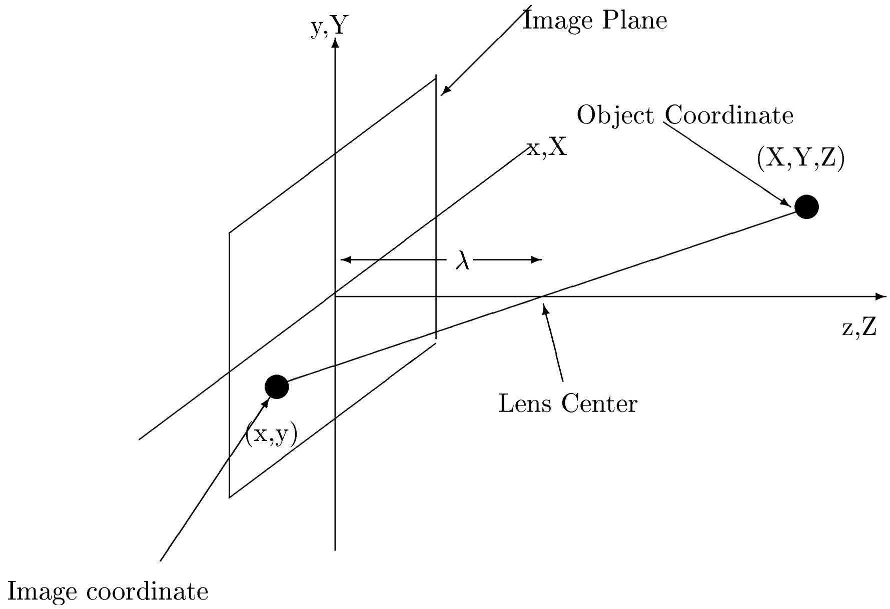

Figure 2: Perspective projection imaging model: Camera coordinate system(x,y,z) is aligned with the Object coordinate system(X,Y,Z)

3 Robust techniques for estimation

Many advances have taken place in the robust control community - mainly in a deterministic setting. It is therefore advantageous to exploit some of these concepts in the estimation problem. This however implies that the framework be deterministic. The Kalman filter deals with uncertainties in a rather ad-hoc fashion since the only way they can be taken care of is through varying Q and R. Quite often "structured" and "unstructured" information about the uncertainty is available; it is difficult to incorporate such knowledge in the synthesis of a Kalman filter in a straightforward manner. We believe that the deterministic setting along with well developed concepts like H_∞ synthesis and μ analysis would provide much more robust estimates to the vision based tracking problem.

From the point of view of implementaion and the fact that measurements are available only at discrete instants of time, the estimator designed using these techniques for the vision problem will be a discrete time one.

In order to design a discrete time filter for this system the continuous time dynamic model is first converted to discrete time system assuming a zero order hold on its inputs. Since the measurement model is nonlinear, we linearize it about the apriori state estimate obtained from the projection of the previous estimate. The linearized C matrix is given by

$$C = \lambda/(\lambda - z) \begin{bmatrix} 1 & 0 & 0 & 0 & -r_1 sin(x_5) & 0 \\ 0 & 0 & 1 & 0 & r_1 cos(x_5) & 0 \\ 1 & 0 & 0 & 0 & -r_2 sin(x_5+\alpha) & 0 \\ 0 & 0 & 1 & 0 & r_2 cos(x_5+\alpha) & 0 \end{bmatrix}$$

The formulation for the H_∞ optimal discrete time estimation problem is as follows

Given a discrete time system:

$$x(k+1) = Ax(k) + Bw(k)$$

$$y(k) = Cx(k) + v(k) \tag{4}$$

it is desired to extract from the corrupted measurements a combination of states

$$\psi = Lx$$

The aim is to design a filter F to produce the estimate of the state combination $\hat{\psi} = L\hat{x}$ such that the supremum of the ratio of the 2-norm of the error to the 2-norm of the disturbance i.e

$$\sup_{\omega, w(\neq 0), v(\neq 0)} \left\{ \frac{||\psi - \hat{\psi}||_2}{||w||_2 + ||v||_2} \right\}$$

is minimized.

4 Implementation Issues

The synthesis of the discrete H_∞ filter was done using the "dhfsyn" command in the μ-Analysis and synthesis Toolbox of MATLAB. One of the requirements for the computation of the filter is that the discrete time system matrix A have no eigen values on the unit circle. Since every constant velocity motion problem generically has an "integration" present in it, we perturbed the A matrix slightly so that the eigen values were inside the unit circle. The resulting filter has a structure similar to the Kalman filter. The H_∞ gain is not directly available. Rather what the function gives us are the A, B, C, D matrices of the filter from which the update gain can be suitably extracted by comparison with the usual A, B, C, D matrices of a current estimator. For the motion problem the gain is a time-varying one.

5 Simulation Results

The performance of the discrete H_∞ filter was compared with that of the Kalman filter for the nominal and perturbed system. The following observations were made

- The performance of the Kalman filter and the H_∞ filter are comparable when no uncertainty is assumed in the system model and apriori state estimates as observed in figure 4 and in other simulation runs
- In the event of a large error in the initial estimate the H_∞ filter estimates converge rapidly to the actual states as compared with the Kalman filter as observed in figure 5 and in other simulations.
- For an uncertainty in the A-matrix of the discrete system which may be due to uncertain sampling period or due to some other unstructured perturbation the performance of the H_∞ filter is superior to that of the Kalman filter. Figure 6 shows the case when there is a 10 % diagonal uncertainty in the continuous time system A matrix. Similarly for a 25% uncertainty in the sampling period the performance of the H_∞ filter was superior to that of the Kalman filter as seen from figure 7. Many other simulations demonstrated the superior robustness of the H_∞ filter.

6 Robustness Analysis of the discrete H_∞ filter

All estimators make use of a nominal system model in estimating the state combination of interest. It is quite natural to expect that this model may not be perfect i.e. some uncertainty may be present in the system model. One of the desirable properties an estimator should possess is that it should be "robust" to such uncertainties. In particular the performance of the filter should not deteriorate significantly in the presence of uncertainty.

An elegant framework namely the Structured Singular Value or μ analysis [9] exists for evaluating stability and performance robustness of feedback control systems. This framework can also be used for the estimation problem on hand.

6.1 Uncertainties in the system model

The uncertainty in the system A matrix can arise due to uncertain sampling period which leads to a real parametric uncertainty. Another source of uncertainty is the presence of unmodeled dynamics which gives rise to complex or real uncertainty. The uncertainty associated with the A matrix is denoted by Δ_A Since the C matrix used by the filter algorithm is based on the local linearization of the nonlinear measurement equations, the uncertainty associated with it is taken to be a full block real-uncertainty(denoted by Δ_C).

The uncertain system model is as shown in figure 3.

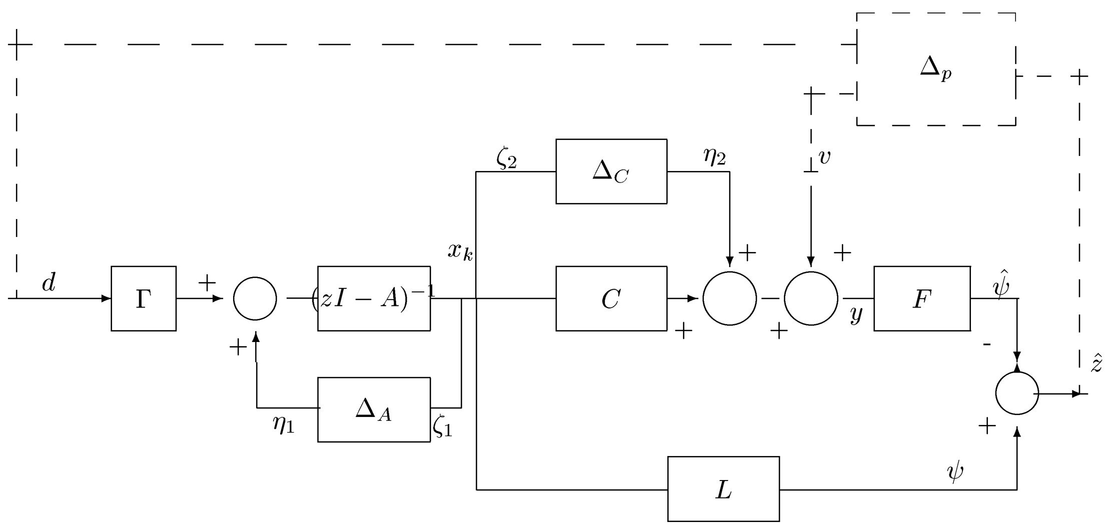

Figure 3: Uncertainties present in the system model

6.2 Performance Robustness analysis using μ

Given the uncertainty structures for the model we can now proceed to compare the performance robustness of the Kalman and H_∞ filter. To do this a fictitious full block uncertainty is assumed between the error and disturbance, denoted by Δ_p together with the actual uncertainty present in the system. We then proceed to build the open loop interconnection structure which involves finding the transfer function of the system between the output and input of each uncertainty block as shown in figure 8. Then the filter (Kalman or H_∞ filter) is introduced in between the y and $\hat{\psi}$ signals.

Once this is done, a μ analysis of the interconnection matrix together with the filter is carried out. For a real diagonal perturbation in the A matrix (which may result from incorrect modeling) and an unstructured complex full block perturbation in the C matrix the μ robust performance plots are as shown in the figure 9.

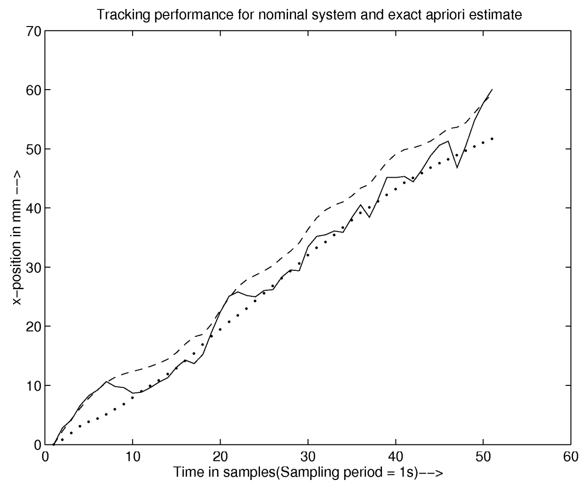

Figure 4: Dotted - Actual, Solid - H_∞ estimate, Dash - Kalman Filter estimate

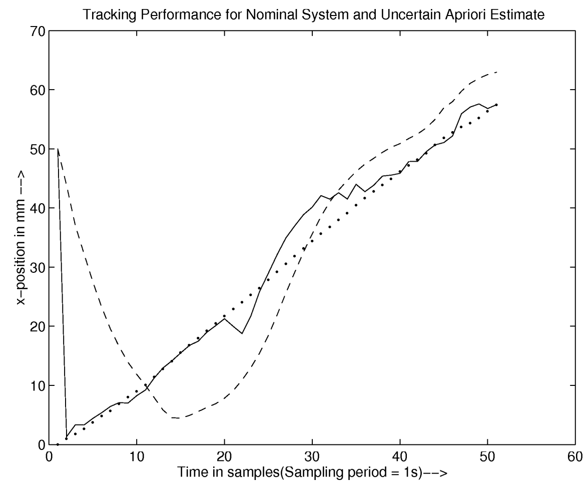

Figure 5: Dotted - Actual, Solid - H_∞ estimate, Dash - Kalman Filter estimate

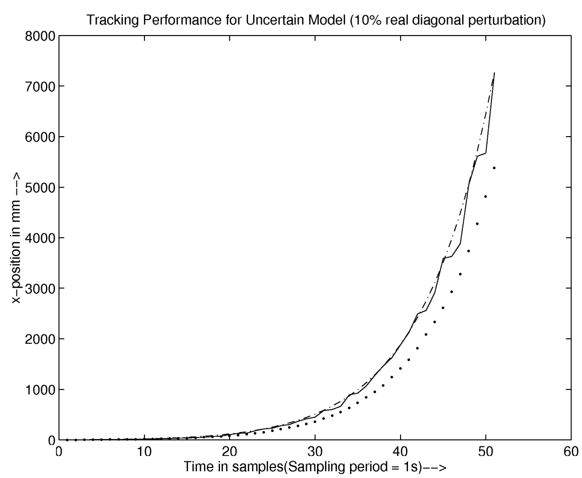

Figure 6: Dash Dotted - Actual, Solid - H_∞ estimate, Dotted - Kalman Filter estimate

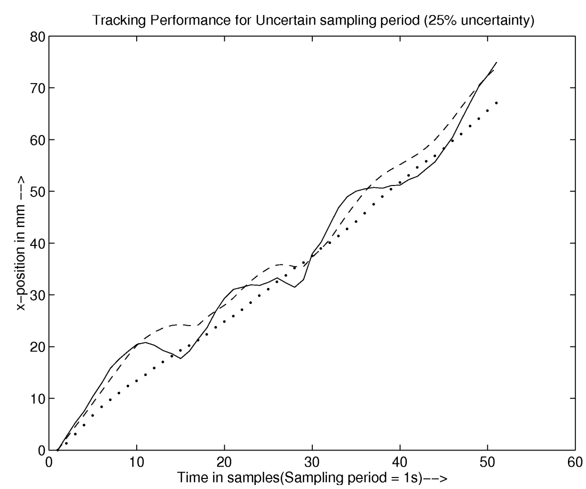

Figure 7: Dotted - Actual, Solid - H_∞ estimate, Dash - Kalman Filter estimate

As is clear from the figure, the peak of the μ plot of the H_∞ filter (denote it by γ_h) lies below that of the Kalman filter (denote it by γ_k). From the robust performance theorem of μ analysis we conclude that there exists an uncertainty block

$$\Delta_w = diag\{\Delta_A, \Delta_C\}$$

of size $1/\gamma_h$ such that the norm of the perturbed disturbance to error transfer function becomes equal to γ_h. This perturbation is referred to as the "worst case" perturbation. From the fact that $\gamma_h < \gamma_k$,

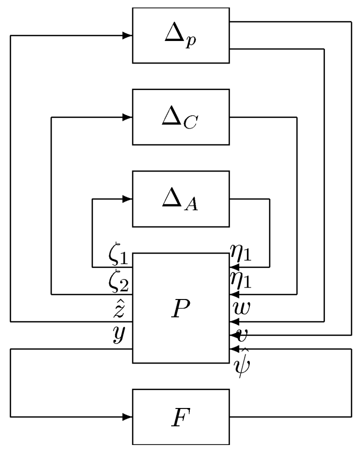

Figure 8: Robustness Performance analysis

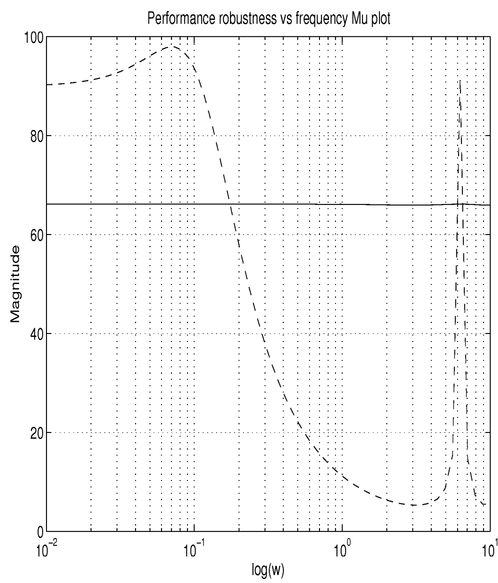

Figure 9: Solid - H_∞Filter, Dashed - Kalman Filter

it can be concluded the performance robustness of the H_∞ filter is greater than the Kalman filter. The simulation results are consistent with the performance robustness results. Similar results were also obtained when a full block complex uncertainty was considered in the A matrix.

7 Conclusions

The objective of the study was to employ the framework of robust control to the motion-parameter estimation problem. As a first step towards this goal we have carried out simulations on a two-dimensional problem with both rotation and translation of the object. The results have been encouraging and the

μ technique has allowed us to absorb the realistic variations in a sound mathematical framework. Two aims in our further endeavors are

- Further refining of the H_∞ filter by using the μ synthesis technique and
- Implementing the technique on real-data obtained from a vision sensing system.

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